Testing the Structural Interpretation of the Price Puzzle with a Cost-Channel Model*

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Abstract

We estimate a variety of small-scale new-Keynesian DSGE models with the cost channel to assess their ability to replicate the ‘price puzzle’, i.e. the inflationary impact of a monetary policy shock typically arising in vector autoregression (VAR) analysis. To correctly identify the monetary policy shock, we distinguish between a standard policy rate shifter and a shock to ‘trend inflation’, i.e. the time-varying inflation target set by the Fed. Our estimated models predict a negative inflation reaction to a monetary policy tightening. We offer a discussion of the possible sources of mismatch between the VAR evidence and our own.

I. Introduction

What is the short-run reaction of inflation to an unexpected and temporary monetary policy tightening? Macroeconomic textbooks suggest that inflation should react negatively to such a monetary policy move (Woodford, 2003a; Gali, 2008). As a matter of fact, however, empirical investigations based on the vector autoregression (VAR) methodology cast doubts on this prediction.

Figure 1 (top panel) depicts the impulse response functions produced with a VAR estimated with post-WWII US data. An unexpected one-shot increase in the policy rate leads to (i) a significantly positive reaction of the policy rate, (ii) a significantly negative reaction of the output gap, and (iii) a significantly positive reaction of inflation. This evidence stands in stark contrast with conventional wisdom. Eichenbaum (1992) labels this evidence...
the ‘price puzzle’ (the ‘VAR evidence’ henceforth). Importantly, Castelnuovo and Surico (2010) show that this result is robust to the implementation of an alternative identification strategy, based on sign restriction, which does not assume recursiveness, and it is then consistent with the timing of models such as the popular standard new-Keynesian framework.

In fact, a possible interpretation of this VAR empirical regularity is offered by models embedding the ‘supply channel’, otherwise known as the ‘cost channel’. The idea is simple. Cash-constrained firms must borrow money from financial intermediaries to pay the wage bills to workers before the goods market opens. Consequently, the interest rate paid on borrowings enters firms’ marginal costs and influences firms’ price setting, so giving a structural role to the presence of the policy rate in the new-Keynesian Phillips curve (NKPC). This creates an extra link between monetary policy moves and aggregate inflation fluctuations (Chowdhury, Hoffmann and Schabert, 2006; Ravenna and Walsh, 2006; Kilponen and Milne, 2007; Surico, 2008; Tillmann, 2009; and Llosa and Tuesta, 2009). Clearly, if the inflationary impact induced by monetary policy moves via the supply channel is stronger than the one operating via the standard ‘demand channel’, a positive reaction of inflation to a monetary policy tightening may very well realize.

The plausibility of such a structural interpretation, however, is ultimately an empirical issue. This article employs Bayesian techniques to estimate a new-Keynesian small-scale DSGE model embedding the cost channel. The model is an extension of the baseline,
aggregate-demand (AD) based set up widely employed to scrutinize US inflation (Benati, 2008; Benati and Surico, 2008; Benati and Surico, 2009; Canova, 2009). The model has the potential to replicate the VAR evidence, because the supply channel may be strong enough to more than compensate the effects on inflation working via the traditional demand channel, a compensation which may induce a (mildly) positive or a muted inflation reaction to a model-consistent monetary policy shock. Our exercise aims exactly to understand if such prediction is actually supported by the data.

Our main results are as follows. First, we do find some statistical support for the cost channel. Due to the presence of the short-term interest rate in the inflation equation, the cost-channel model fits the data better with respect to the standard ‘demand channel only’ new-Keynesian framework. Second, we do reject the structural interpretation of the price puzzle. Clearly, the data prefer a parameterization of the model for which the demand channel is relatively stronger than the cost channel in transmitting the monetary policy impulses to inflation. In the small-scale model we work with, the estimated degree of interest rate smoothing is the main ingredient which boosts the demand side’s relative strength. Possibly, this is so because inflation expectations are strongly influenced by a gradual monetary policy conduct (Woodford, 2003b). Then, an increase in marginal costs driven by drifts in the policy rate is less inflationary than it would be under a less persistent policy conduct. Our results are robust to a variety of perturbations of the baseline analysis – model specification, measures of the output gap, sample selection. We then conclude that the VAR price puzzle is not a fact, but instead an artefact possibly due to VAR misspecification.

Before moving to the next section, we make contacts with some strictly related contributions. Barth and Ramey (2001) analyse different US sectors and find that sectorial differences in the working capital may rationalize the heterogeneous impact across sectors of a monetary policy shock. Similar results are obtained by Gaiotti and Sacchi (2006) for Italy, and Dedola and Lippi (2005) for France, Germany, Italy and the United Kingdom. In a single-equation framework, Ravenna and Walsh (2006) support the presence of the cost channel for the US economy, Tillmann (2008) for the United States, UK and Euro Area, and Chowdhury et al. (2006) for Canada, France, Italy and the United States. There is then support for the empirical relevance of the cost channel for a variety of countries, the United States being among them.

As regards the structural interpretation of the VAR evidence, Chowdhury et al. (2006) couple an estimated Phillips curve embedding the cost channel with a calibrated demand side, and show that such model qualitatively replicates the VAR evidence as for Italy, the UK and the United States. Christiano et al. (2005) estimate a model featuring several nominal and real rigidities by indirect inference (impulse response matching), and also replicate such a fact. With the same econometric strategy, Henzel et al. (2007) obtain similar results for the Euro Area. While offering stimulating results, this evidence is not conclusive. Calibrated models may lead to dynamics that are at odds with the data. Then, the ‘cost channel interpretation’ must be tested by taking models to the data. Indirect inference conducted by matching VAR impulse responses assumes the VAR price puzzle to be ‘true’, instead of ‘testing’ it. We then prefer to conduct our econometric exercise with likelihood-bases techniques, which are free to reject the VAR evidence if the data prefer so.

The article closest to ours is probably Rabanal (2007). He investigates the sign and magnitude of the inflation reaction to a monetary policy shock by estimating a medium-scale
model a la Christiano, Eichenbaum and Evans (2005) with Bayesian techniques, and finds evidence supporting the ‘textbook’ monetary policy transmission mechanism. In his article, the key drivers for this result are (i) a less than full wage indexation, (ii) a moderate wage stickiness, and (iii) a high price stickiness. Our article differs from Rabanal’s (2007) along several dimensions. First, we jointly model the standard monetary policy shock and the shock to ‘trend inflation’, i.e. the time-varying inflation target set by the Fed. This is a key modelling choice. In fact, if the Fed had actually pursued a time-varying inflation target, in assuming a constant target we would force the dynamics of the inflation target to enter the ‘residual’ of the policy rule, and we would label as ‘policy shock’ what, de facto, is a convolution of the true policy innovation and the inflation target dynamics. Bache and Leitemo (2008) show that this misspecification can dramatically bias impulse responses to a monetary policy shock in autoregressive models. Given the empirical evidence pointing towards trend inflation in the United States (Ireland, 2007; Cogley and Sbordone, 2008; Cogley, Primiceri and Sargent, 2010), we believe the separate identification of these two monetary policies shocks to be quite relevant for the issue at stake. Second, we relax the unitary upper bound to the cost-channel parameter imposed by Rabanal (2007) when conducting his estimates. While such an upper bound represents a natural imposition when interpreting the cost channel parameter as the share of financially constrained firms in the economy, frictions on the financial markets may indeed suggest a pass through from the policy rate to the lending rate larger than one (Chowdhury et al., 2006; Tillmann, 2008). Moreover, model misspecification – e.g. a too simplistic banking sector – may easily turn the structural cost-channel parameter to a reduced form capturing the direct effect of the policy rate to inflation. At that point, however, the imposition of a unitary upper bound would not necessarily be warranted. For these reasons, we allow the cost-channel parameter to take values above one, so possibly strengthening the supply channel. In conducting our investigation, we estimate a fairly large battery of semi-structural frameworks, which turn out to invariably reject the hypothesis of a supply channel being stronger than the textbook demand channel. Our contribution should be seen as complementary with respect to Rabanal’s (2007).

The article develops as follows. Section II presents the benchmark new-Keynesian model with the cost channel we work with. Section III documents and discusses our empirical findings. In section IV, we provide further evidence by investigating different subsamples, employing proxies for the output gap, scrutinizing alternative (semi)structural frameworks, and conducting a variance decomposition analysis. In section V, we discuss the drivers of the deflationary reaction to a monetary policy shock. Section VI proposes a brief literature review on the reasons underlying the possible misspecification of the policy shock in VAR analysis. Section VII concludes.

II. A model with the cost channel

Structure of the model

Our benchmark model reads as follows:

\[ \text{The variables in the model are expressed in log deviations with respect to their non-stochastic steady state values or, as for output, in deviations with respect to its long-run trend.} \]
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\[
\pi_t = \frac{\beta}{1 + \alpha \beta} E_t \pi_{t+1} + \frac{\alpha}{1 + \alpha \beta} \pi_{t-1} + \kappa [ (\sigma + \eta) x_t + \psi R_t ] + \epsilon_t^\pi, \tag{1}
\]

\[
x_t = \frac{1}{(1 + h)} E_t x_{t+1} + \frac{h}{(1 + h)} x_{t-1} - \frac{(1 - h)}{\sigma (1 + h)} (R_t - E_t \pi_{t+1}) + \nu_t^x, \tag{2}
\]

\[
R_t = \phi_R R_{t-1} + (1 - \phi_R) [ \phi_x (\pi_t - \pi^*_t) + \phi_x x_t ] + \nu_t^R, \tag{3}
\]

\[
\pi^*_t = \rho * \pi^*_{t-1} + \epsilon_t^*, \tag{4}
\]

\[
\nu_t^* = \rho * \nu^*_{t-1} + \epsilon_t^*, \tag{5}
\]

\[
\epsilon_j^j \sim \text{i.i.d. } N(0, \sigma_j^2), \quad j \in \{ \pi, x, R, * \}. \tag{6}
\]

Equation (1) is an expectational NKPC in which \( \pi_t \) stands for the inflation rate, \( \beta \) identifies the discount factor, \( \alpha \) indicates price indexation to past inflation, \( x_t \) identifies the ‘output gap’ – whose impact on current inflation is influenced by the slope-parameter \( \kappa \) (a convolution of the discount factor and the probability of non-reoptimizing prices by firms), \( \sigma \) is the representative consumers’ degree of relative risk aversion, \( \eta \) is the inverse of the labour supply elasticity and \( \epsilon_t^\pi \) is interpreted as ‘inflation’ shock, or ‘supply’ shifter. Differently with respect to the ‘demand channel only’ models equation (1) embeds a direct impact of the nominal interest rate \( R_t \) on the inflation rate, which is active as long as the ‘cost channel’ parameter \( \psi \) is positive. To be clear, the presence of the cost channel is, in the framework we deal with, a necessary but not sufficient condition to obtain a ‘price puzzle’, which arises conditional on a subset of all the possible calibrations of the structural parameters of the model we focus on.

Equation (2) is obtained by log linearizing the consumption Euler equation stemming from the household’s intertemporal problem. Output fluctuations are driven both by expectations on future realizations of the output gap and by the \textit{ex-ante} real interest rate, whose loading (the intertemporal elasticity of substitution, i.e. IES) is a convolution of habits and relative risk aversion. The demand shock \( \nu_t^x \), which is autoregressive as suggested by equation (5), is interpreted as households’ preference shock or a fiscal shock. Equation (3) is a Taylor rule postulating the systematic reaction of the policy rate to movements in the inflation gap and the output gap. Past policy decisions matter, and their impact is captured by the interest rate smoothing parameter \( \phi_R \), as in Clarida, Galí and Gertler (2000). The zero-mean i.i.d. random shock \( \epsilon_t^R \) stands for the monetary policy innovation. The evolution of the inflation target – formalized by equation (4) – is dictated by the autoregressive parameter \( \rho^* \) as well as the volatility \( \sigma^* \) of its innovation \( \epsilon_t^* \). This process is typically assumed to be a random walk or a very persistent variance-stationary process capturing the low-frequency component of the inflation rate, which are likely to be sensible approximations of the time-varying target set by monetary-policy authorities. The innovation processes (6) close the model.

A set up similar to the one hereby presented (time-varying inflation target aside) has recently been object of theoretical investigations by Chowdhury \textit{et al.} (2006); Ravenna
and Walsh (2006); Kilponen and Milne (2007); Surico (2008); Tillmann (2009); and Llosa and Tuests (2009).

III. Empirical analysis

We estimate the models (1)–(6) with Bayesian methods (see An and Schorfheide, 2007; Canova and Sala, 2009) for the sample 1954:III–2008:II, US quarterly data. We limit our study to the second quarter of 2008 so as to avoid dealing with the acceleration of the financial crises began with the bankruptcy of Lehman Brothers in September 2008, which triggered non-standard policy moves by the Fed (Brunnermeier, 2009). Importantly, the use of a full system approach is likely to limit the weak instruments problem affecting GMM when applied to hybrid schedules displaying rational expectations among the drivers of the modelled variables (Mavroeidis, 2004; Canova and Sala, 2009). Moreover, the full system estimation enables us to account for cross-equation restrictions clearly affecting the estimation of NKPC’s parameters (for a maximum likelihood application to the Euro-area NKPC, see Fanelli, 2008). In the model we focus on, a notable example regards the time-varying inflation target, which enters (also) the solution of the inflation rate and shapes its persistence, so clearly affecting the estimate of the cost-channel parameter as well as others (e.g. price indexation).

To estimate the model, we employ three observables. The output gap is computed as percentualized log deviation of the real GDP with respect to the potential output as computed by the CBO.3 The inflation rate is the quarterly growth rate of the GDP deflator.4 Finally, for the short-term nominal interest rate we consider the effective federal funds rate (averages of monthly values) expressed in quarterly terms. The source of the data is the Federal Reserve Bank of St Louis (FREDII). All the transformed data are demeaned before estimation. This choice is a result of the fact that, in steady state, all the variables of our model assume zero value. This is clearly at odds with the facts, and forces us to prefilter the data so to allow for a meaningful match between the model and the data. However, this is hardly crucial for the result. In fact, when re-estimating the model with undemeaned data and ad hoc, series-specific constants, we obtain the very same results we document in the article.

Priors

Our Bayesian estimation calls for the imposition of prior densities on the model parameters. First and foremost, we have to set a prior for the cost-channel parameter $\psi$. Exploring US data with Bayesian techniques, Rabanal (2007) estimates it to be 0.15 in a full sample analysis, and 0.56 for the 1980s and 1990s. Ravenna and Walsh (2006) appeal to single-equation GMM estimation and find it to be 1.276 (benchmark estimate of the battery they provide), a value very close to that put forward by Chowdhury et al. (2006), who propose 1.3 on the basis of GMM estimation. Christiano et al. (2005) set it to 1. In order not to play against the cost-channel interpretation of the inflation reaction found with VARs, we

3Ravenna and Walsh (2006) show that the deviation from a flexible price equilibrium is not an adequate measure of output gap in a model with an active cost channel. We estimate models with real unit labour cost (ULC) as empirical proxy for the output gap in section 4.

4Our results are robust to the employment both the personal consumption expenditures and consumer price index deflators (see Castelnuovo, 2009).
assume $\psi$ to be normally distributed with mean 1.75, a value larger than the the estimates surveyed here. However, to remain relatively agnostic on this parameter, we allow for a fairly large standard deviation, i.e. 0.7. Notice that, following economic intuition, we impose a zero lower bound to the domain of our prior density (we return on this issue in the next subsection). We also impose dogmatic priors as for the inverse of the labour supply elasticity $\eta$ and the slope of the NKPC. As for the former parameter, since we do not employ labour data in the estimation, and given the identification issues regarding it, we calibrated it to 1, a standard value in the literature. Preliminary attempts to estimate the slope of the NKPC led to implausibly low realizations, a problem encountered by e.g. Ireland (2004). We then set $\kappa$ to 0.05, a value in line with recent empirical evidence (Benati and Surico, 2008; Benati and Surico, 2009).

As for the trend inflation process, we follow Cogley et al. (2010) and set the autoregressive parameter $\rho_\tau$ to 0.995 to force the trend inflation process to capture low-frequency movements in inflation. Following the convention, we also fix the discount factor $\beta = 0.99$ (corresponding to an annual discount rate of approximately 4%). The remaining priors are standard, and in line with Benati and Surico (2008), Benati and Surico (2009) and Cogley et al. (2010) as for the parameters in common. Table 1 collects our prior densities.

### TABLE 1

Bayesian estimates of the benchmark model. Full sample and subsample posterior densities

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>$N(1.75, 0.7)[0, 10]$</td>
<td>1.18 [0.60, 1.75]</td>
<td>1.01 [0.22, 1.74]</td>
<td>1.12 [0.36, 1.84]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$\beta(0.5, 0.285)$</td>
<td>0.01 [0.00, 0.02]</td>
<td>0.02 [0.00, 0.05]</td>
<td>0.01 [0.00, 0.03]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$N(1, 0.05)$</td>
<td>0.89 [0.36, 1.02]</td>
<td>0.94 [0.36, 1.02]</td>
<td>0.94 [0.36, 1.02]</td>
</tr>
<tr>
<td>$h$</td>
<td>$\beta(0.7, 0.15)$</td>
<td>0.78 [0.71, 0.86]</td>
<td>0.68 [0.56, 0.80]</td>
<td>0.82 [0.74, 0.90]</td>
</tr>
<tr>
<td>$\phi_\tau$</td>
<td>$N(1.7, 0.3)$</td>
<td>1.87 [1.49, 2.24]</td>
<td>1.70 [1.31, 2.96]</td>
<td>1.91 [1.54, 2.31]</td>
</tr>
<tr>
<td>$\phi_\gamma$</td>
<td>$\Gamma(0.3, 0.2)$</td>
<td>0.72 [0.45, 1.01]</td>
<td>0.47 [0.25, 0.69]</td>
<td>0.61 [0.36, 0.91]</td>
</tr>
<tr>
<td>$\phi_\rho$</td>
<td>$\beta(0.5, 0.285)$</td>
<td>0.93 [0.91, 0.95]</td>
<td>0.90 [0.86, 0.94]</td>
<td>0.94 [0.92, 0.96]</td>
</tr>
<tr>
<td>$\rho_\tau$</td>
<td>$\beta(0.5, 0.285)$</td>
<td>0.40 [0.27, 0.52]</td>
<td>0.45 [0.29, 0.62]</td>
<td>0.52 [0.38, 0.68]</td>
</tr>
<tr>
<td>$\sigma_\tau$</td>
<td>$\Gamma(0.1, 0.25)$</td>
<td>0.43 [0.36, 0.49]</td>
<td>0.51 [0.38, 0.65]</td>
<td>0.22 [0.16, 0.28]</td>
</tr>
<tr>
<td>$\sigma_\rho$</td>
<td>$\Gamma(0.1, 0.25)$</td>
<td>0.25 [0.21, 0.30]</td>
<td>0.34 [0.26, 0.42]</td>
<td>0.21 [0.16, 0.26]</td>
</tr>
<tr>
<td>$\sigma_\gamma$</td>
<td>$\Gamma(0.1, 0.25)$</td>
<td>0.22 [0.21, 0.24]</td>
<td>0.19 [0.16, 0.21]</td>
<td>0.14 [0.12, 0.15]</td>
</tr>
<tr>
<td>$\sigma_\phi$</td>
<td>$\Gamma(0.1, 0.25)$</td>
<td>0.05 [0.03, 0.08]</td>
<td>0.06 [0.03, 0.09]</td>
<td>0.04 [0.03, 0.06]</td>
</tr>
<tr>
<td>Log(ML)</td>
<td>—</td>
<td>-423.64</td>
<td>-229.62</td>
<td>-79.84</td>
</tr>
<tr>
<td>Log(ML$</td>
<td>_{\psi=0}$)</td>
<td>—</td>
<td>-425.59</td>
<td>-227.78</td>
</tr>
</tbody>
</table>

**Notes:** Prior densities: figures indicate the (mean, SD) of each prior distribution. The prior domain for the cost-channel parameter is constrained to be [0, 10]. Posterior densities: figures reported indicate the posterior mean and the [5th, 95th] percentile of the estimated densities. Details on the estimation procedure provided in the text. Marginal likelihoods computed via Laplace approximation. ML, marginal likelihood.
Posterior densities and Bayesian impulse responses

Given the vector $\xi = (\psi, \beta, \alpha, \beta, \phi, \phi_R, \phi_x, \phi_{Rt}, \phi_x, \phi, \beta^*, \sigma, \sigma_x, \sigma_R, \sigma_x^*)$ of structural parameters, the vector of endogenous variables $z_t = [x_t, \pi_t, R_t]'$, the autoregressive demand shock $\epsilon_t = [\epsilon_t^a]'$, the vector of innovations $\eta_t = [\epsilon_t^a, \epsilon_t^h, \epsilon_t^R, \epsilon_t^x]'$, and the vector of observable variables we aim at tracking $Y_t = [x_t, \pi_t, R_t]'$, we write the model in state space form, we relate the latent processes to the observable variables via the measurement equation (without assuming any measurement errors), and we employ the Kalman filter to evaluate the posterior densities and Bayesian impulse responses.

Our posterior estimates are reported in Table 1 (first column).

Posterior densities and Bayesian impulse responses

Our posterior estimates are reported in Table 1 (first column). First, we focus on the cost-channel parameter $\psi$. Its posterior mean reads 1.18, clearly smaller than the prior mean – the latter being 1.75. Moreover, the fifth percentile of its posterior density reads 0.60, a value clearly larger than zero. Importantly, this result does not appear to be driven by our prior choice. In line with economic intuition, we assume a prior with positive mean to capture the possible increase in firms’ marginal costs following an interest rate hike. However, the reduced form flavour of our cost-channel parameter, and possibly model misspecification, might in principle call for a negative value of such parameter. We then re-estimate the model with the prior $\psi \sim N(0, 1)$, and impose no lower bounds. Interestingly, we find that the marginal likelihood remains de facto unaffected ($-423.93$). As for the cost-channel parameter, its posterior mean (90% credible set) is 0.88 ([0.25, 1.44]), slightly smaller than the benchmark estimate. Importantly, when shutting the cost channel down, the marginal likelihood deteriorates, and the Bayes factor amounts to $\exp(-423.93 - (-425.59)) \approx 5.26$. This deterioration offers ‘positive’ evidence in favour of the cost-channel model. The posterior mean is close to the point estimates put forward by Ravenna and Walsh (2006) and Chowdhury, et al. (2006), and it is somewhat larger than the one by Rabanal (2007).

The remaining estimated parameters assume values in line with previous contributions (e.g. Rabanal, 2007; Smets and Wouters, 2007; Justiniano and Primiceri, 2008a). In particular, the NKPC turns out to be purely forward looking, a finding recently discussed, among others, by Benati (2009), Cogley and Sbordone (2008) in a NKPC in which trend inflation appears as a further driver. Kleibergen and Mavroeidis (2009) perform GMM estimation with identification-robust methods of the semistructural version of the NKPC we focus on (cost channel aside), and cannot reject the null of purely forward looking inflation process. The demand shock is fairly persistent, but the estimated autoregressive

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6Estimations obtained by adding white noise measurement errors – not shown for the sake of brevity, but available upon request – confirmed the robustness of our findings.

7Some technical details on our estimation strategy are confined in our Appendix.

8We computed the log-marginal likelihoods both by means of the Laplace approximation around the posterior mode (based on a normal distribution) and via the modified harmonic mean estimator (Geweke, 1998), which exploits the draws from the posterior distribution. The two methods deliver virtually identical results. This is caused by the close-to-normal distribution of all the estimated posteriors. Given the large computational gains implied by the Laplace approximation, we employ this approximation for our model comparison.

9According to Kass and Raftery (1995), a Bayes factor between 1 and 3 is ‘not worth more than a bare mention’, between 3 and 20 suggests a ‘positive’ evidence in favour of one of the two models, between 20 and 150 suggests a ‘strong’ evidence against it, and larger than 150 ‘very strong’ evidence.

10Technically, Benati (2009) and Cogley and Sbordone (2008) consider NKPC curves log linearized around a positive value for the inflation rate in steady state, i.e. ‘trend inflation’ as popularized by Ascari (2004). Differently, we consider a model consistent with a zero inflation rate in steady state.
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51.0 1 5
–0.8
–0.6
–0.4
–0.2
0
0.2
51.0 1 5
–1
–0.5
0
0.1
0.2
51.0 1 5
–0.1
0
0.1
0.2
0.3
1982:IV–2008:II

Figure 2. Bayesian impulse response functions to a monetary policy shock. Solid blue lines: mean impulse response. Dotted blue lines: 5th and 95th percentiles of the posterior distributions. Red circles: 5th and 95th percentiles of the impulse responses computed by calibrating the model with our priors, i.e. no data involved in the computation of these responses. Simulations employed to compute the red circled responses: 10,000. Shock size normalized so to induce a 25 basis point-jump of the quarterly policy rate

parameter value is far from unity. This suggests that in the model there is a propagation mechanism of the shocks capable to replicate the unit-root like dynamics of output without the need of imposing a unit-root (or almost unit-root) IS disturbance.

Ultimately, however, this analysis aims at pinning down the reaction of inflation to a monetary policy shock. Figure 2 (top panel, blue lines) displays the estimated dynamics responses to such a shock. The data speak clearly. The presence of an active cost channel is far from overturning conventional wisdom, in that a monetary policy tightening opens a recession, and such downward demand pressure leads to a statistically significant deflationary phase. The output gap then follows an hump-shaped convergence pattern towards the steady state. The policy rate and the inflation rate also gradually go back to their steady states.

Impulse response functions: the role of priors

It is of interest to assess the impact of our prior densities on the estimated parameters. Too tight priors could very well predetermine our findings in favour of the demand channel. To gain information in this respect, we contrast our estimated impulse response with the 5th and 95th percentiles of the densities obtained by (i) sampling 10,000 realizations from our prior densities, (ii) calibrating the parameters of our model accordingly, and (iii) computing the impulse responses to a normalized monetary policy shock. Figure 2 (red circles) depicts
these impulse responses. Some considerations are in order. First, our set of priors imply a quite uncertain on-impact reaction of quarterly inflation to a monetary policy shock, which range from (mildly) positive values to negative values close to $-0.5$. This range is substantially larger than that implied by our Bayesian estimated reactions, which very precisely indicate a negative reaction of about $-0.2$. The shrinkage of the 90% credible set is clearly driven by the data, which turn out to be substantially informative. Second, the demand channel, conditional on our priors, is admittedly much stronger than the cost channel. Indeed, it is hard to produce a spectacular positive reaction of inflation to a monetary policy shock with this small-scale model. Chowdhury et al. (2006) calibrate a restricted version of the models considered here for the US economy, and obtain a muted reaction of inflation to a monetary policy shock. Interestingly, Rabanal’s (2007) model is clearly more successful along this dimension. Possibly, this is because of the frictions he accounts for in his analysis, which are just unmodelled here. In particular, in the model he employs, inflation depends on marginal costs including the real wage, the rental rate of capital, and the nominal interest rate. Staggered wage setting with indexation makes the response of the real wage (to a monetary policy shock) smoother, while a high variability in the capital utilization rate makes the response of the rental rate of capital less volatile. This implies that the hike in the nominal interest rate followed by a monetary policy shock is not counterbalanced by drops in real wages and the rental rate of capital in the short-run, and an inflationary reaction caused by relatively strong(er) cost channel may emerge. In contrast, our framework features flexible wages and absence of physical capital. Consequently, the cost channel is somewhat weaker, and the demand channel tends to prevail.11

This being acknowledged, our priors suggest that mildly positive or muted responses of inflation and output could, in principle, very well emerge out of our econometric exercises. However, the data clearly work against this prediction, and call for a negative inflation reaction also with a model with time-varying trend inflation and a reduced-form cost-channel parameter whose estimated value may go over 1. Rabanal (2007), with a more sophisticated model, verifies that the necessary conditions to obtain a price puzzle are just rejected by the data. Then, the conclusion against the superiority of the supply channel is robust to the employment of two different frameworks.

While being statistically significant, the economic role of the cost channel appears to be present but limited. Figure 3 displays the impulse response of the benchmark model to the four estimated shocks, and contrasts them with those estimated in the ‘no cost channel’ scenario. Indeed, differences appear to be marginal, with the exception of the reaction of the output gap to a trend inflation shock, which is clearly milder when the cost channel is considered, possibly because of more moderate interest rate deviations from the steady state. These responses suggest that the relevance of the cost channel is actually conditional to the type of shock a researcher is interested in.

11 A note on the calibration of the (inverse of the) labour elasticity is warranted. Our benchmark calibration is $\eta = 1$. Under this calibration, the labour supply is upward sloping with respect to the real wage, then an output contraction – triggered by a monetary policy shock – causes a fall in labour demand and real wages that negatively affects inflation. In contrast, the (extreme) calibration $\eta = 0$ would call for an infinitely elastic supply with respect to the real wage, with the latter being independent hours. Hence, there would be no effect of the variation of hours worked on the real marginal costs, and the ‘price puzzle’ would be more likely to arise. In fact, when repeating our exercise under $\eta = 0$, we verified a slight increase of the probability of modelling the ‘price puzzle’ with our structural model. However, such a calibration has no appreciable effect of our conclusions drawn on the basis of our estimated frameworks.
IV. Diagnostic and further investigations

Subsample stability

The analysis developed so far has relied on the assumption of stability of the structural parameters in the sample at hand, as in Smets and Wouters (2007) and Justiniano and Primiceri (2008a). However, the appointment of Paul Volcker as Chairman of the Fed, occurred in August 1979, has been associated to a break in the US monetary policy conduct (Clarida et al., 2000; Lubik and Schorfheide, 2004; Boivin and Giannoni, 2006; Benati and Surico, 2009; Mavroeidis, 2010). To control for this break, we re-estimate the model by focusing on the subsamples 1954:III–1979:II and 1982:IV–2008:II. We do not include the span 1979:III–1982:III not to deal with the ‘Volcker experiment’, i.e. the period during which Chairman Paul Volcker targeted non-borrowed reserves, a monetary policy hard to describe with a Taylor rule.

Our results are displayed in Table 1 (second and third columns) and Figure 2 (second and third rows). The two main messages are robust to this subsample analysis. First, there is an active cost channel, whose importance is supported by the marginal likelihood comparison in the second subsample. In fact, the first subsample is less supportive, but it is still hard to clearly reject the cost channel’s importance. Second, the effect of a monetary policy tightening is clearly deflationary. As regards the remaining parameters, one may notice that the systematic policy reaction to inflation gap fluctuations is larger in the second subsample, a finding in line with several recent studies (Lubik and Schorfheide, 2004; Boivin and Giannoni, 2006; Benati and Surico, 2009; Cogley et al., 2010; Mavroeidis, 2010; Castelnuovo, 2011).
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2010c). In the first subsample, such estimated reaction is larger than what is typically found in the literature. This is because of the fact that we truncate the parameter space and concentrate on the ‘determinacy territory’. Moreover, in this model the object targeted by the Fed is the inflation gap, as opposed to the raw inflation rate typically considered in Taylor rule estimations – notable exceptions being Castelnuovo, Greco and Raggi (2008) and Cogley et al. (2010) The estimated shocks’ standard deviations are clearly smaller in the 'Great Moderation’ subsample, a finding already put forward by Justiniano and Primiceri (2008b). The remaining parameters, cost-channel parameter included, display stability over subsamples, with the exception of habit formation, which increases.

**Autocorrelation functions**

It is important to check if the model adequately captures the dynamics in the data. We then study the autocorrelation functions of the estimated (smoothed) shocks. Unfortunately, as shown in Figure 4 (top panel), the performance of the benchmark model is not entirely satisfactory. The demand and inflation target shocks are clearly autocorrelated. The cost-push shock and, above all, the monetary policy shock display realizations inconsistent with the white noise hypothesis. This evidence cast doubts on our results, in that shocks mis-specification, above all that concerning the monetary policy shock, might in principle lead to a distorted representation of the dynamics of the system. We then estimate an ‘enriched’ framework featuring (i) an ARMA(1,1) representation of the cost-push shock, which is in line with the price markup shock estimated by (Smets and Wouters, 2007); (ii) an autoregressive monetary policy shock, in line with (Rudebusch, 2002); and (iii) an ECM for the inflation target, for which we estimate – instead of calibrating – the persistence parameter. Formally, we modify the following subset of equations of our benchmark model:

\[
\pi_t = \frac{\beta}{1 + \alpha \beta} E_t \pi_{t+1} + \frac{\alpha}{1 + \alpha \beta} \pi_{t-1} + \kappa (\sigma + \eta) x_t + \psi R_t + v_t^\pi, \tag{7}
\]

\[
v_t^\pi = \rho v_{t-1}^\pi + \epsilon_t^\pi - \gamma x_t v_{t-1}^\pi, \tag{8}
\]

\[R_t = \phi_R R_{t-1} + (1 - \phi_R) [\phi_x (\pi_t - \pi_t^*) + \phi_x x_t] + v_t^R, \tag{9}
\]

\[v_t^R = \rho v_{t-1}^R + \epsilon_t^R, \tag{10}
\]

\[\pi_t^* = \rho \pi_{t-1}^* - \gamma (\pi_{t-1} - \pi_{t-1}^*) + \epsilon_t^*, \tag{11}
\]

A note on the encompassing inflation target process (11) is warranted. Policymakers may gradually change their inflation target because of their evolving knowledge of the

---

12Clarida et al. (2000), Lubik and Schorfheide (2004), Boivin and Giannoni (2006), and Mavroeidis (2010) offer support to the ‘indeterminacy’ hypothesis to explain the US macroeconomic dynamics in the 1970s. Castelnuovo and Surico (2010) show that indeterminacy may offer a rationale for the price puzzle typically found when estimating the effects of a monetary policy shocks with VAR models. Surico (2006) discusses the perils coming from merging two subsamples featuring different equilibria. However, Sims and Zha (2006), Justiniano and Primiceri (2008b) and Cogley et al. (2010) cast doubts on multiple equilibria as a relevant feature to describe the dynamics of the 1960s and 1970s. Moreover, Castelnuovo (2010b) shows that the equilibrium selection strategy one implements under indeterminacy may importantly drive the model-consistent theoretical volatilities. We then decided to stick to the uniqueness scenario.
transmission mechanism, or their time-varying preferences for the inflation-output stabilization trade-off. In doing so, they may refer to realized inflation to adjust their inflation target. If past inflation was (say) over the target, policymakers may signal their intentions of bringing inflation back to the target by lowering the target itself. Rational agents will recognize the Fed’s intentions, and will adjust their expectations accordingly.\textsuperscript{13} Of course, this is a ‘testable hypothesis’. If the reduced form parameter $\gamma$ in equation (11) is estimated to be non-zero, then the just described adjustment is supported by the US data. In contrast, if $\gamma = 0$, we are back to the exogenous inflation target process proposed by, among others, Ireland (2007) and Cogley \textit{et al.} (2010).\textsuperscript{14}

We estimate this alternative ‘enriched’ model composed by the equations (2) (5)–(11). Table 2 (first column of results) reports the prior densities of all the estimated parameters along with the posterior means and the corresponding [5th, 95th] posterior percentiles. Some considerations are in order. First, attempts to estimate the cost-channel parameter conditional on this enriched structure led the mode optimizer to point towards the zero

\textsuperscript{13}In the United States, during the sample we focus on, the target was not explicitly communicated to the public. However, the Fed has communicated and commented its past decisions extensively, as well as launched signals concerning possible future decisions – e.g. via numerous speeches by the Fed’s Governors.

\textsuperscript{14}We verified with a set of simulations that, instead, a negative value for the $\gamma$ parameter would lead the system to a number of explosive roots inferior with respect to the number of forward looking variables, i.e. indeterminacy, with high probability (which also depends on the calibration of the remaining structural parameters of the model).
lower bound imposed by our constrained normal a priori. We then fixed this parameter to zero, and re-estimated the model. Second, the moving-average coefficient of the cost-push shock ARMA(1, 1) model is estimated to have a posterior mean equal to 0.19, but the 90% credible set comprises the zero value. Third, the ‘ECM’ for the inflation target displays an ECM coefficient \( \gamma \) whose posterior mean is equal to 0.69, while that of the persistence parameter \( \rho_e \) equal to 0.18, much lower than the calibration proposed by Cogley et al. (2010). Fourth, this enriched structure suggests quite different posterior means for a number of parameters (with respect to the benchmark estimates), including habit formation and the policy parameters, which are estimated to have lower posterior means, and the persistence of the demand shock, which is estimated to be higher. Fifth, the policy shock is found to be extremely persistent. Rudebusch (2002) states that the smooth behaviour of the policy rate observed in the United States (and a variety of other countries) is not intention-

<table>
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<tr>
<th>Param.</th>
<th>Prior dens.</th>
<th>( \psi )</th>
<th>( K )</th>
<th>( \kappa )</th>
<th>( h )</th>
<th>( \phi )</th>
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<th>Log(ML)</th>
<th>Log(ML) ( \mid \rho = 0 )</th>
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<td>( N(1.75, 0.7)) ( (0, 10) )</td>
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<td>( -350.22 )</td>
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<td>( \Gamma(0.05, 0.01) )</td>
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<td>( \kappa )</td>
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<td>( h )</td>
<td>( \beta(0.7, 0.15) )</td>
<td>( 0.28 )</td>
<td>( 0.92 )</td>
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<td>( \phi )</td>
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<td>( 1.73 )</td>
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| Notes: Prior densities: Figures indicate the (mean, SD) of each prior distribution. The prior for the cost-channel parameter is constrained to be [0, 10]. Posterior densities: Figures reported indicate the posterior mean and the [5th, 95th] percentile of the estimated densities. Details on the estimation procedure provided in the text. Marginal likelihoods computed via Laplace approximation.

CBO, Congressional Budget Office; ULC, unit labour cost; ML, marginal likelihood.
ally implemented by the Fed, but it is instead caused by serially correlated monetary policy shocks. While lending support to Rudbusch’s conjecture, our estimates are also consistent with policy gradualism, i.e. interest rate smoothing. Indeed, when estimating a restricted version of the model with $\phi_R = 0$, we observe a deterioration of the marginal likelihood of about $\exp(2.5) \approx 12$ points. This finding corroborates previous research by English, Nelson and Sack (2003) and Castelnuovo (2003), who support the hypothesis of gradualism intentionally pursued by the Fed in the post-WWII sample. Sixth, the autocorrelation functions suggest that these model manipulations induce (close to) white-noise processes for the estimated shocks of the enriched model. This can be seen by looking at Figure 4 (bottom panel). The autocorrelation functions of the demand and inflation target shocks are consistent with the null hypothesis of white noise process. As for the cost-push and monetary policy shocks, signs of autocorrelation are milder than in the benchmark case. We can then infer that the enriched model fits the data better. This conclusion is corroborated also by the estimated marginal likelihood, which suggests a spectacular difference of about 90 log-points.

What does this model predict in terms of inflation reaction to a monetary policy tightening? Given that the cost-channel parameter is set to zero, the model obviously predicts a negative reaction of inflation. Figure 5 (top panel) reports the estimated reactions for the three variables of interest. The on-impact reaction of inflation is three times larger than suggested by the benchmark model. Inflation is quite persistent, and the convergence towards the steady state is very slow and smooth. The same holds true as for the policy rate, which ‘overshoots’ and takes negative values for most of the observed span, and for the output gap.

Real ULC

In our exercise, the output gap is measured as log deviations of the real GDP with respect to the CBO potential output. In other words, it is constructed as log deviation of the real GDP with respect to its trend. While being a very widely adopted measure of output gap in empirical work (see, among others, Benati, 2008; Benati and Surico, 2008; Benati and Surico, 2009), this measure does not match the ‘theoretically relevant’ measure of output gap, which is instead captured by the real ULC. In fact, as pointed out by Gali and Gertler (1999), ULC correlates negatively with a variety of measures of detrended output. This is true also in our sample: the CBO output gap and ULC have a correlation equal to $-0.33$. Moreover, Ravenna and Walsh (2006) show that the deviation from the flexible price equilibrium is not the ‘welfare-relevant’ measure of output gap in a model with the cost channel.

Therefore, two issues arise. First, which are the consequences of employing CBO output gap versus ULC as empirical proxies (indicators) of the business cycle? Second, which business cycle indicator does the Fed react to? The first issue in an exquisitely empirical one. In line with Canova (1998), different representations of the business cycle may lead to different estimates of relevant objects such as the impulse response functions to a structural shock. The second issue has solid theoretical grounds, in that a central bank may focus on the stabilization of output around its ‘efficient’ level, which does not necessarily coincide with its trend. We tackle the first issue in this subsection, and leave the second issue to the next subsection.

Given that our conclusions may in principle be severely distorted by the employment of the wrong empirical proxy for the output gap, we conduct an extensive investigation to
Figure 5. Bayesian impulse response functions: alternative frameworks. ‘Enriched, CBO’: model with ARMA(1,1) cost-push shock, AR(1) monetary policy shock, error-correction mechanism for the inflation target. ‘Benchmark, ULC’: benchmark structure consistent with real unit labour costs (ULC) as proxy for the output gap. ‘Reduced form, CBO’: model structure as ‘benchmark, ULC’, but with the Congressional Budget Office (CBO) output gap used as proxy for the output gap. Further details in the text.

scrutinize this issue. First, we replace the CBO output gap with ULC and rerun the trivariate VAR with which we estimate the effects of a monetary policy shock. Figure 1 (bottom row) plots the estimated impulse responses. Two considerations are in order. First, the positive reaction of inflation to a monetary policy shock is extremely robust to this perturbation of the VAR in terms of shape and magnitude. Hence, the evidence in favour of the price puzzle is clearly still there. The reaction of the policy rate is also basically unaffected. Second, the reaction of the output gap is significantly positive, i.e. the negative correlation between detrended output and ULC is not only unconditional, but also conditional to a monetary policy shock identified with Cholesky restrictions. This is an interesting finding, because it proposes a new puzzle, i.e. the ‘output gap puzzle’. Further evidence, not shown for the sake of brevity but available upon request, suggests that this puzzle is robust to the inclusion of the CBO output gap on top of the ULC indicator – when the former is also considered, its reaction is significantly negative, but those of the inflation rate and the ULC are significantly positive at a 68% confidence level. We postpone the discussion on the possible interpretation of this VAR fact to section VI.

We then estimate a version of the structural DSGE model with the cost channel by using ULC as observable. According to the theory (Gali and Gertler, 1999), we consider the following version of the NKPC:

\[ \begin{align*}
\pi_t &= \gamma_1 \pi_{t-1} + \gamma_2 y_t + \delta u_t + \varepsilon_t, \\
\end{align*} \]
Testing the structural interpretation of the price puzzle

\[ \pi_t = \frac{\beta}{1 + x_\beta} E_t \pi_{t+1} + \frac{x}{1 + x_\beta} \pi_{t-1} + \kappa(x_t + \psi R_t) + v_t^*, \]  

which enables us not to impose any dogmatic prior for \( \kappa \) and \( \eta \). We then estimate the ‘benchmark, ULC’ model composed by equations (2)–(6) and (12). Table 2 (second column) reports our estimates. As for the cost-channel parameter, we obtain a posterior mean equal to one, and a 90% credible set [0.30, 1.65]. When contrasting the ‘benchmark, CBO’ model (Table 1, first column) with the ‘benchmark, ULC’ model (Table 2, second column), one may notice differences in the posterior means of most of the parameters. Also, the estimated slope of the Phillips curve in the ULC model is 0.02, a value in line with the estimates put forward by Galí and Gertler (1999).

What we are mostly interested in, however, is the reaction of the inflation rate to a monetary policy shock. Figure 5 – central row – displays the Bayesian responses conditional on the estimated ‘enriched, ULC’ model. The 90% credible set of both inflation and output features negative values in the short run. Then, contrarily to what is suggested by the SVAR in Figure 1, the model-consistent reaction of inflation and output to a monetary policy shock is negative. This result may be because of the different proxy for the output gap employed in this exercise (ULC, as opposed to CBO output gap), or to the different structure of the NKPC (equation (12), as opposed to the previously considered equation (1)). To disentangle these two effects, we re-estimate the models (2)–(6) and (12) with the CBO output gap (which replaces ULC). Given that we are not imposing the (possibly too taxing) theoretical restrictions coming from microfoundations, we term this model ‘reduced form, CBO’. Figure 5 (bottom row) displays the estimated impulse responses as for this model. It is immediate to notice that they are extremely similar to those implied by the ‘benchmark, ULC’ model. Indeed, as documented in Table 2 (second and third columns), most of the estimated parameters of these two models feature very similar posterior means. Interestingly, while the ULC model does not lend formal support to the presence of the cost channel, the CBO model’s marginal likelihood does so, with a deterioration of the marginal likelihood under \( \psi = 0 \) of about \( \exp(7) \) points, which represent very striking evidence in favour of the cost channel. Interestingly, the marginal likelihood of this ‘reduced form, CBO’ model is much higher than that of the benchmark framework’s, the difference being about 80 log-points.

Which indicator of the business cycle does the Fed react to?

As pointed out in the previous subsection, one should be careful in modelling the reaction of policymakers to business cycle indicators.\textsuperscript{15} If we believe that policymakers aim at stabilizing output around its trend, then the model should be estimated using the CBO output gap indicator (as done in the previous sections). If we instead believe that policymakers understand that some fluctuations in output are the efficient response of the economy to business cycle shocks, then they should stabilize output around a measure of potential output different from the trend—the ‘efficient’ level of output. In the new-Keynesian model, the gap between real output and its efficient level is a linear transformation of ULCs.

If the latter case is the one empirically supported by the data, we should find a positive and significant estimate for the parameter \( \phi_x \) in the Taylor rule with ULC, while not

\textsuperscript{15}I thank an anonymous referee for stimulating me to write this subsection.
necessarily so as for the policy rule with the CBO output gap. Without the presumption of being exhaustive as for this quite relevant discussion, we estimate some simple Taylor rules to shed some light on this issue. Given that the discussion revolves around which indicator should enter the Taylor rule, we follow Clarida et al. (2000) and undertake a single-equation estimation in which the Fed is assumed to react to raw inflation and a business cycle indicator in a gradual fashion. This single equation approach is ‘robust’, in the sense that it allows to circumvent the perils related to model misspecification typically arising when working with a full-system estimation technique. We therefore contrast the least-square estimates obtained with the CBO output gap with those conditional on the employment of ULC.

Our results as for the CBO output gap (sample: 1954:III–2008:II) read as follows:

\[
R_t = 0.12 \pi_t + 0.03 x_{CBO}^t + 0.93 R_{t-1} + \hat{\nu}_R^t, \\
\hat{\nu}_R^t = 0.16 \hat{\nu}_R^{t-1} + \hat{\epsilon}_R^t, \hat{\sigma}_{\hat{\epsilon}} = 0.21, \bar{R}^2 = 0.93,
\]

where */**/*** indicate significance at the 90/95/99% confidence level.\(^{16}\)

With ULC, instead, we obtain:

\[
R_t = 0.12 \pi_t - 0.05 x_{ULC}^t + 0.92 R_{t-1} + \hat{\nu}_R^t, \\
\hat{\nu}_R^t = 0.18 \hat{\nu}_R^{t-1} + \hat{\epsilon}_R^t, \hat{\sigma}_{\hat{\epsilon}} = 0.22, \bar{R}^2 = 0.93.
\]

Some comments are in order. First, the estimated reaction of policymakers to the CBO output gap is positive and significant.\(^{17}\) This result lines up with our estimates obtained with Bayesian techniques.

Second, our estimated Taylor rule returns a significantly negative value as for the coefficient capturing the reaction to ULC, our empirical proxy for the welfare-relevant indicator. This sign is inconsistent with our prior. Notably, our least-square point estimate is different in sign and magnitude with respect to the one obtained with Bayesian techniques. This discrepancy puts in evidence the impact of our priors as regards our full-system estimation.

How to interpret our least-square results based conditional on ULC? One interpretation is that ULC is not a good empirical approximation of the welfare-relevant output gap. While being possible, this is unlikely in light of the microfoundations of such measure offered by the widely scrutinized new-Keynesian framework. Another interpretation is the possibly genuine policymakers’ attention put on a measure like the CBO output gap (or measures positively correlated to it), as opposed to the welfare-relevant ULC output gap, which is arguably more difficult to estimate.

Third, the contribution of the business cycle indicator to the fit of the policy rate is negligible. No matter what measure of output we focus on, the adjusted \(R^2\) is equal to 0.93, and it is barely unchanged when estimating constrained versions of the policy rule under \(\phi_x = 0\) (results not shown for the sake of brevity). This suggests that \(\phi_x\) is hardly a key

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\(^{16}\)Our statement is conditional on the object of interest for the policymakers being a constant transformation of our relative ULC empirical measure.

\(^{17}\)Heteroskedasticity-consistent standard errors obtained with the white correction for the estimated variance-covariance (VVC matrix).
parameter as for the impulse responses to a monetary policy shock in our estimated model (for further investigations on this issue, see Castelnuovo, 2009).

Fourth, the estimates of $\phi_n$ and $\phi_R$ are significant and extremely stable across different business cycle indicators. Again, this seems to suggest that, while being theoretically relevant, the issue on which concept of output the Fed has focused its attention on is of limited empirical relevance as far as the estimation of the monetary policy shock is concerned.

These results are robust across subsamples. In particular, ULC induces the wrong sign of the parameter $\phi_n$ also when the subsamples 1954:III–1979:II and 1982:IV–2008:II are employed. The CBO output gap, instead, takes the correct sign and is significant also as for these two subsamples. This evidence points towards the CBO output gap as the more likely indicator the Fed has reacted to in the post-WWII US sample. Moreover, no matter what the output definition targeted by the Fed is, our main result – the negative reaction of inflation to monetary policy shock in the estimated model – is hardly driven by such object.

**Forecast error variance decomposition**

A key point is the identification of the drivers of inflation dynamics. What is the share of inflation explained by the transmission mechanism as opposed to its own cost-push shock? Indeed, the persistence displayed by our impulse responses is in principle consistent with most of the volatility of inflation being driven by its own disturbance. If this were true, the cost channel would have little space to show up and explain much of the business cycle.

To investigate this issue further, we compute the forecast error variance decomposition of inflation at two different horizons, i.e. ‘medium run’ (16-step ahead) and ‘long run’ ($\infty$-step ahead). Table 3 reports our findings. First, the variance decomposition of inflation is

<table>
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<th>TABLE 3 Forecast error variance decomposition of inflation</th>
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<tr>
<td>$\epsilon_x$</td>
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<tr>
<td>Benchmark</td>
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<td>$\infty$-step ahead</td>
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<td>Enriched, CBO</td>
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<tr>
<td>Reduced form, CBO</td>
</tr>
<tr>
<td>16-step ahead</td>
</tr>
<tr>
<td>$\infty$-step ahead</td>
</tr>
</tbody>
</table>

*Notes: ‘Enriched, CBO’: model with ARMA(1, 1) cost-push shock, AR(1) monetary policy shock, ECM for the inflation target. ‘Benchmark, ULC’: benchmark structure consistent with real unit labour costs (ULC) as proxy for the output gap. ‘Reduced form, CBO’: model structure as ‘benchmark, ULC’, but with the CBO output gap used as proxy for the output gap. Further details in the text. Figures computed by relying on posterior modes.*
clearly model specific. Perhaps not surprisingly, models with a highly persistent inflation target assign a very large role to inflation target shocks, with figures over 70%. In contrast, the ‘enriched, CBO’ framework assigns a negligible weight to such disturbance, and a much higher weight to policy shocks, above all in the long run. Second, also the explanatory power of ‘non-policy, demand’ shocks is limited, with the ‘benchmark’ model predicting, however, about 12% of the medium run forecast error volatility being explained by such shocks. Most importantly, the cost-push shock is the most important driver of inflation volatility in none of the scenarios under investigation. The ‘enriched, CBO’ scenario is the most favourable scenario to such shock, with a 45% associated to the medium run horizon. However, also in this case the policy shock displays a (slightly) superior explanatory power (48%), which becomes clearly larger when moving to the long run horizon (63%, vs. the cost-push shock’s 32%). Interestingly, it is exactly in this scenario that the demand channel turns out to be mostly supported (recall the evidence provided in Figure 5, top row). Then, while giving the cost channel some chances to arise, our empirical investigations simply rebut it as a possible reason behind the VAR price puzzle evidence.

A note on the role of trend in inflation shocks is warranted. Such shocks emerge as main drivers of inflation in three models of four, not only in the long run (a predictable outcome, given our calibration of the persistence parameter of the trend inflation shock), but also in the medium run. In the light of the possibly severe distortions induced by the study of impulse responses to a monetary policy shock in presence of time-varying trend inflation (Bache and Leitemo, 2008), our findings corroborate our choice of modelling explicitly the trend inflation shock and distinguish its effects from those coming from the ‘traditional’ monetary policy shock $\varepsilon^R$.

V. Understanding the result

We have established that, empirically, the cost channel is not sufficient to induce a positive (or, at least, muted) response of inflation to a monetary policy shock. Which are the drivers of this result? To answer this question, we concentrate on the theoretically sound and empirically relevant ‘benchmark, ULC’ framework composed by equations (2)–(6) and (12).

Suppose to shut down all the sources of persistence which may affect the impulse response of inflation to a monetary policy shock in this model. Then, it is easy to show that, under very plausible conditions on a set of parameters of the model, the necessary and sufficient condition to obtain an inflationary effect out of a monetary policy shock is $\psi - \sigma^{-1} > 0$ (see our Appendix for the derivation). This condition clearly pins down the tension between the supply and demand channels. The stronger the cost channel, the more likely the inflationary effect ceteris paribus. On the other hand, a higher IES, not surprisingly, increases the likelihood of a deflation to occur.

As a matter of fact, this condition is close to be met by our estimates. Let us stick to the ‘benchmark, ULC’ model. The cost-channel parameter’s posterior mean is 0.99, while the risk aversion is 1.02. Hence, 0.99 − 1/1.02 ≈ −0.01. Then, why do we get a clearly negative inflation reaction? To gauge some intuition, Figure 6 plots the impulse responses of the ‘benchmark, ULC’ model along with those of several restricted versions of it, estimated by switching off $\phi_R, h, \zeta$ and $\phi_s$ one at a time. Interestingly, quite an evident impact
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Figure 6. Impact of parameter restrictions on impulse response functions. ‘Benchmark, ULC’: benchmark structure consistent with real unit labour costs (ULCs) as proxy for the output gap. Further details in the text

on the inflation reaction comes from switching interest rate smoothing off. Indeed, while being mildly negative on impact, the inflation reaction is, de facto, muted. This implies that the supply channel is, in relative terms, weakened by a gradually implemented monetary policy. Possibly, this is so because inflation expectations are strongly influenced by interest rate smoothing (Woodford, 2003b), then the demand channel is stronger when some policy inertia is allowed for. Indeed, the data clearly rebut the ‘no interest rate smoothing scenario’. Given the marginal likelihood of the ‘benchmark, ULC’ case, which is equal to $-351.28$, the Bayes factor concerning the comparison between such model and the restricted version with $\phi_R = 0$ reads $\exp(-351.28 - (-537.11)) \approx 5.0692 \exp(80)$, a very large number indeed! Interestingly enough, the other panels suggest that the influence of habit formation, price indexation and the policy reaction to output, while being present, is actually very mild. Then, according to this small-scale model, interest rate smoothing importantly regulates the relative strength of the demand (as opposed to supply) channel.

Of course, one may force the model to obtain an inflationary effect out of a policy shock. To do so, we fix $\psi = 2$ (twice its point estimate, see Table 2 (second column)) and $\phi_R = 0$, and re-estimated the model. Figure 6 (bottom panel) displays the outcome of this exercise. While being very mild, the on-impact inflation reaction is positive, i.e. the ‘price puzzle’, at least in terms of sign, is captured. But this comes at a very high cost in terms of model fit. In fact, the Bayes factor reads $\exp(-351.28 - (-545.61)) \approx 2.4914 \exp(84)$, an extremely large figure.
VI. VAR misspecification and the price puzzle

Our exercise leads to a rebuttal of the structural interpretation of the VAR evidence. Rabanal (2007) reaches the same conclusion by focusing on the importance of different sources of persistence in the Christiano et al. (2005) model. Then, if estimated models do not offer a rationale for the price puzzle, why do we observe it in VARs?

Sims (1992) was the first to point out that the price puzzle is likely to be a result of a misspecification of the monetary policy shock. In fact, if the central bank reacts to expected inflation, then a predicted upcoming surge in inflation will be followed by an increase in the policy rate, a decrease in the output gap, and – as long as the monetary policy tightening is not such to fully offset the inflationary shock – a rise in current inflation. If the VAR omits expected inflation, and if expected inflation and current inflation are not strictly linked (i.e. current inflation is not a ‘sufficient statistic’ for expected inflation), then the supposed-to-be monetary policy shock in a trivariate VAR in inflation, output gap, and policy rate will somewhat naturally capture the positive correlation between inflation and the policy rate, i.e. it will produce a price puzzle. Sims (1992) proposed to add an indicator of nascent inflation (commodity prices) to the vector of variables of interest. While not solving the price puzzle problem, this trick clearly renders the picture less puzzling.

The omitted variable issue is also tackled by Bernanke, Boivin and Eliasz (2005), Boivin, Giannoni and Mihov (2009), and Bork, Dewachter and Houssa (2009), who show that by allowing some factors extracted by a large panel of variables to enter the VAR (as ‘endogenous variables’) the price puzzle tends to disappear. Forni and Gambetti (2010) focus on an open economy VAR and show that both the price puzzle and the forward discount puzzle – which refers to the small-scale VAR evidence of a ‘delayed overshooting’ – disappear when a data-rich approach in the context of a structural factor model is considered. Castelnovo and Surico (2010) show that the price puzzle evidence is actually limited to the pre-Volcker subsample – similar evidence is provided by Barth and Ramey (2001), Hanson (2004) and Boivin and Giannoni (2006). Working with a model in which they simulate a policy shift resembling the one estimated for the US case, Castelnovo and Surico (2010) show that a standard trivariate VAR estimated on pseudo-data may indeed produce a price puzzle when, in fact, the model generating such pseudo-data suggests a negative inflation reaction to a policy tightening. They show that this is possibly because of the omission in the VAR of inflation expectations under the weak monetary policy scenario.

Interestingly, some of the best predictors of future inflation turn out to be useless for correcting the bias in the dynamics of inflation, as shown by Hanson (2004). Other recent contributions have pointed towards other types of VAR misspecifications. Leeper and Roush (2003) show that money is important for well specifying the monetary policy shock when studying economies in which a double-causal link between money and interest rate might have occurred. In particular, if the central bank reacts contemporaneously to monetary aggregates, and if money demand is contemporaneously driven by the nominal interest rate, then the omission of money would lead to a misspecification of the monetary policy shock. A different issue is raised by Giordani (2004), who shows that the omission of potential output in standard trivariate VARs may severely bias impulse responses and be the responsible of the price puzzle. In fact, potential output appears in all the equations of a standard new-Keynesian AD/AS model. Hence, its omission will lead supposed-to-be
shocks to be residuals correlated across the VAR equations, and consequently to produce biased impulse response functions. Romer and Romer (2004) stick on a standard trivariate VAR but produce a careful measure of the monetary policy shocks based on changes in the intended federal funds rate and the Fed’s expectations on future inflation and output. Such new measure of monetary policy shock does not imply any price puzzle in their estimated VARs.

In a recent paper, Carlstrom, Fuerst and Paustin (2009) show that a price puzzle may actually arise if a Cholesky-identification scheme is used to identify the VAR policy shock when, in fact, such restriction is wrong. Carlstrom et al. (2009) present a battery of scenarios which, under different calibrations of a structural model employed as data generating process, feature different responses of inflation and output produced with an estimated Cholesky-SVAR. Interestingly, one of these cases is actually the ‘price puzzle’ – ‘output gap puzzle’ depicted in Figure 1 (bottom panel). Hence, the price puzzle, and puzzles arising out of the employment of SVARs with shocks identified with zero restrictions in general, may be actually caused by the imposition of wrong restrictions. Castelnuovo (2010a) offers empirical support to Carlstrom et al.’s (2009) conjectures.

While presenting somewhat different views on how to model a monetary policy shock in a VAR framework, these articles clearly express a common view on the ‘price puzzle’, i.e. they qualify it as an ‘artefact’ because of model misspecification, more than a genuine ‘fact’.

**VII. Conclusions**

This article has shown that a new-Keynesian model embedding the cost channel may hardly offer a rationale for the price puzzle typically found when conducting VAR analysis. Under some particular parameterizations of the model, a positive inflation reaction to an unexpected, restrictive monetary policy may actually arise. However, when taking the model to the data, the structural interpretation of the VAR evidence is clearly rebutted. The impact exerted by the estimated systematic monetary policy gradualism is shown to possibly drive this result, at least conditional on small-scale models like the ones we deal with. Our findings are robust to several perturbations to the baseline analysis, including different sample selection, model specifications, and specifications of the monetary policy shocks. We think of this result as being important for understanding the sign (and the magnitude) that monetary policy actions should optimally undertake in response to shocks moving inflation off target.

We stress that this article does offer some evidence in favour of the cost channel. In particular, the presence of such channel seems to be economically important when assessing the reaction of output to a trend inflation shock. In general, the structural role of the interest rate in the Phillips curve calls for a serious rethinking of optimal monetary policy in presence of supply effects. Contributions along this path have recently been proposed by Ravenna and Walsh (2006) and Kilponen and Milne (2007). Moreover, given the uncertainty surrounding the magnitude of the cost-channel parameter, more research is needed both for the quantification of the importance of the cost-channel and for the design of an optimal monetary policy in presence of cost-channel uncertainty, an issue tackled by Tillmann (2009). Llosa and Tuesta (2009) analyse the relationship between uniqueness
and learnability of equilibrium in presence of supply effects, a topic of great relevance for policymakers. We plan to participate to this exciting agenda with further investigations in the close future.

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### Appendix

#### Bayesian estimation

To perform our Bayesian estimation we employed DYNARE, a set of algorithms developed by Michel Juillard and collaborators, and freely available at [http://www.dynare.org/](http://www.dynare.org/).

The model is estimated by implementing a two-step strategy. First, we estimate the mode of the posterior distribution by maximizing the log-posterior density, which combines our priors on the parameters of interest with the likelihood function. Second, we employ the random-walk Metropolis–Hastings algorithm to estimate the posterior distribution. The mode of each parameter’s posterior distribution was computed by using the ‘csminwel’ algorithm elaborated by Chris Sims. A check of the posterior mode, performed by plotting the posterior density for values around the computed mode for each estimated parameter...
in turn, confirmed the goodness of our optimizations. We then exploited such modes for initializing the random walk Metropolis–Hastings algorithm to simulate the posterior distributions. In particular, the inverse of the Hessian of the posterior distribution evaluated at the posterior mode was used to define the VCV matrix of the chain. The initial VCV matrix of the forecast errors in the Kalman filter is set to be equal to the unconditional variance of the state variables. We initialized the state vector in the Kalman filter with steady-state values. We simulated two chains of 500,000 draws each, and discarded the first 50% as burn-in. To scale the VCV matrix of the random walk chain we used factors implying an acceptance rate belonging to the [23%, 40%] interval. We verified the convergence towards the target posterior distribution via the Brooks and Gelman (1998) convergence checks.

As typically done in the literature, we discarded all the draws not implying a unique equilibrium of the system. Notably, in presence of supply-side effects, the standard Taylor principle does not apply anymore. For a discussion of the uniqueness conditions in the models (1)–(3) with no endogenous persistence of any sort and with a central bank just reacting to inflation fluctuations, see Brueckner and Schabert (2003). Regarding this issue, Surico (2008) investigates the role played by the systematic reaction to output gap fluctuations. Llosa and Tuests (2009) study the effects of the cost channel on determinacy and learnability of the rational expectations equilibrium.

**Derivation of the inflation rate in equilibrium conditional on a policy shock**

*Proposition.* Given the models (2)–(6) and (12) under $\alpha = h = \phi_z = \phi_r = 0$, the necessary and sufficient condition (under a ‘plausible’ model calibration) to obtain a positive inflation reaction to a monetary policy shock is:

$$\psi > \sigma^{-1}.$$  

*Proof.* Given that we are studying a monetary policy shock, all other shocks in the model remain at their unconditional mean, which is zero. Then, the only shock driving the economy is the monetary policy shock $\epsilon^R_t$, which is a white noise. Consequently, we can guess the solutions $\pi_t = a\epsilon^R_t$ and $x_t = b\epsilon^R_t$. Given the nature of the policy shock, these solutions imply $E_t\pi_{t+1} = E_tx_{t+1} = 0$. Hence, the model simplifies as follows:

$$\pi_t = \kappa(x_t + \psi R_t),$$
$$x_t = -\sigma^{-1} R_t,$$
$$R_t = \phi_x \pi_t + \epsilon^R_t.$$  

By plugging the IS and Taylor rule schedules into the NKPC, it is easy to derive the following expression.

By plugging this expression into the Phillips curve and performing some manipulations, one may easily come up with the closed form solution

$$\pi_t = \frac{\kappa(\psi - \sigma^{-1})}{1 + \kappa(\sigma^{-1} - \psi)\phi_x} \epsilon^R_t,$$

which verifies the guess $\pi_t = a\epsilon^R_t$, and clearly enables us to verify the guess $x_t = b\epsilon^R_t$. 

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Figure 7. Denominator of the ‘price puzzle’ condition: empirical density. See Appendix

It is very safe to state that, under a ‘plausible’ model calibration, $1 + \kappa (\sigma^{-1} - \psi) \phi_x > 0$. Figure 7 plots the empirical density of the expression $1 + \kappa (\sigma^{-1} - \psi) \phi_x$ conditional on 10,000 different sets of sampled values from the posterior densities of the ‘benchmark, ULC’ scenario. Clearly, this inequality is met. Conditional on this result, and given that $\kappa \in \mathbb{R}^+$, the inflation reaction to a monetary policy shock (tightening) is positive iff $\psi - \sigma^{-1} > 0$. 

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