Appendix of the paper "Modest Macroeconomic Effects of Monetary Policy Shocks During the Great Moderation: An Alternative Interpretation" [not for publication]

1 Smets and Wouters (2007) model

We propose here a short presentation of the Smets and Wouters (2007) framework.

The Smets and Wouters (2007) model features a number of nominal and real frictions which are relevant to replicate the features of the main U.S. macroeconomic series. In particular, it models sticky nominal price and wage settings that allow for backward-looking inflation indexation; habit formation in consumption; investment adjustment costs; variable capital utilization and fixed costs in production. The stochastic dynamics is driven by seven structural shocks, namely a total factor productivity shock, two shocks affecting the intertemporal margin (risk premium shocks and investment-specific technology shocks), two shocks affecting the intratemporal margin (wage and price mark-up shocks), and two policy shocks (exogenous spending and monetary policy shocks).

In this model, households maximize a nonseparable utility function in consumption and labor over an infinite life horizon. Consumption appears in the utility function in quasi-difference form with respect to a time-varying external habit variable. Labor is differentiated by a union, so there is some monopoly power over wages, which results in explicit wage equation and allows for the introduction of sticky nominal wages à la Calvo (1983). Households rent capital services to firms and decide how much capital to accumulate given the capital adjustment costs they face. The utilization of the capital stock can be adjusted at increasing cost. Firms produce differentiated goods, decide on labor and capital inputs, and set prices conditional on the Calvo model. The Calvo model in both wage and price setting is augmented by the assumption that prices that are not reoptimized are partially indexed to past inflation rates. Prices are therefore set in function of current and expected marginal costs, but are also determined by the past inflation rate. The marginal costs depend on wages and the rental rate of capital. Similarly, wages depend on past and expected future wages and inflation. The model features, in both goods and labor markets, an aggregator that allows for a time-varying demand elasticity depending on the relative price. Further details can be found in the original paper by Smets and Wouters (2007).
The log-linearized version of the DSGE model around its steady-state growth path reads as follows:

\begin{align*}
y_t &= c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g \\
c_t &= c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} + c_2 (l_t - E_t l_{t+1}) - c_3 (r_t - E_t \pi_{t+1} + \varepsilon_t^b) \\
i_t &= i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \varepsilon_t^i \\
q_t &= q_1 E_t q_t + 1 + (1 - q_1) E_t r_t^{k} - (r_t - E_t \pi_{t+1} + \varepsilon_t^b) \\
y_t &= \phi_p (\alpha k_t^a + (1 - \alpha) l_t + \varepsilon_t^a) \\
k_t^s &= k_{t-1} + z_t \\
z_t &= z_1 r_t^k \\
k_t &= k_1 k_{t-1} + (1 - k_1) i_t + k_2 z_t \\
\mu_t^P &= \alpha (k_t^s - l_t) + \varepsilon_t^a - w_t \\
\pi_t &= \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^P + \varepsilon_t^\pi \\
r_t^k &= -(k_t - l_t) + w_t \\
\mu_t^w &= w_t - (\sigma_t l_t + (1 - \lambda/\gamma)^{-1} (c_t - \lambda/\gamma c_{t-1})) \\
w_t &= w_1 w_{t-1} + w_2 (E_t w_{t+1} + E_t \pi_{t+1}) - w_3 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w \\
r_t &= \rho r_{t-1} + (1 - \rho) (r_\pi \pi_t + r_\gamma (y_t - y_t^g)) + r_{\Delta y} [(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \varepsilon_t^R \\
\varepsilon_t^x &= \rho_x \varepsilon_{t-1}^x + \eta_t^x, x = (b, i, a, R) \\
\varepsilon_t^g &= \rho_g \varepsilon_{t-1}^g + \rho_g^a \eta_t^a \\
\varepsilon_t^z &= \rho_z \varepsilon_{t-1}^z + \eta_t^z - \chi_z \eta_{t-1}, z = (p, w) \\
\eta_t^R &\sim N(0, \sigma_R^2) \\
\end{align*}

where variables endowed with a time subscript are log-deviations from the steady-state, and variables without such subscript are at the steady state. The log-linearized aggregate resource constraint (1) features the convolutions \( c_y = 1 - g_y - i_y, g_y \) and \( i_y \), which stand for the steady-state exogenous spending-output ratio and investment-output ratio. In particular, \( i_y = (\gamma - 1 + \delta) k_y \), where \( \gamma \) is the steady-state growth rate, \( \delta \) is the depreciation rate of capital, \( k_y \) is the steady-state capital-output ratio, and \( z_y = R_y^{k} k_y \) is the steady-state rental rate of capital. Notice that eq. (16), the one of the stochastic processes of the government spending, allows for the productivity shock to affect the log-linearized level of output \( y_t \). This is so because exogenous spending, in this model, includes net exports, which may be affected by domestic pro-
ductivity development. As for the consumption Euler equation (2), \( c_1 = \frac{1}{\gamma} \left( 1 + \frac{1}{\gamma} \right)^{-1} \), 
\( c_2 = (\sigma_c - 1) \frac{W^a L^b}{C^c} \left[ \sigma_c (1 + \frac{1}{\sigma_c}) \right]^{-1} \), and \( c_3 = \left( 1 - \frac{1}{\gamma} \right) \left[ (1 + \frac{1}{\gamma}) \sigma_c \right]^{-1} \). Current consumption \( c_t \) is a function of past and expected future consumption, of expected growth in hours worked, of the ex ante real interest rate, and of a disturbance term \( c_t^b \). Under the assumption of no habits (\( \lambda = 0 \)) and that of log-utility in consumption (\( \sigma_c = 1 \)), \( c_1 = c_2 = 0 \), and we go back to the standard, fully forward-looking Euler equation for consumption. The disturbance term \( c_t^b \) represents a wedge between the interest rate controlled by the central bank and the return on assets held by the households. A positive shock to this wedge increases the required return on assets held by the households. At the same time, it increases the cost of capital and it decreases the value of capital and investment (see below). This is basically a shock very similar to a net-worth shock. This disturbance is assumed to follow a standard AR(1) process. The dynamics of investment \( i_t \) is captured by the investment Euler equation (3), where 
\( i_1 = (1 + \beta \gamma^{1-\sigma_c})^{-1} \) and \( i_2 = [1/(1 + \beta \gamma^{1-\sigma_c}) \gamma^2 \varphi] \), where \( \varphi \) is the steady-state elasticity of the capital adjustment cost function, and \( \beta \) is the discount factor applied by households. Notice that capital adjustment costs are a function of the change in investment, rather than its level. This choice is made to introduce additional dynamics in the investment equation, which is useful to capture the hump-shaped response of investment to various shocks. In this equation, the stochastic disturbance \( c_t^i \) represents a shock to the investment-specific technology process, and is assumed to follow a standard first-order autoregressive process. The value-of-capital arbitrage equation (4) suggests that the current value of the capital stock \( q_t \) depends positively on its expected future value (with weight \( q_1 = \beta \gamma^{-\sigma_c} (1 - \delta) \)), as well as on the expected real rental rate on capital \( E_t r_{t+1}^k \) and on the ex ante real interest rate and the risk premium disturbance. Eq. (5) is the first one of the supply side block. It describes the aggregate production function, which maps output to capital (\( k_t^z \)) and labor services (\( l_t \)). The parameter \( \alpha \) captures the share of capital in production, and the parameter \( \phi_p \) is one plus the share of fixed costs in production, reflecting the presence of fixed costs in production. Eq. (6) suggest that the newly installed capital becomes effective with a one-period delay, hence current capital services in production are a function of capital installed in the previous period \( k_{t-1} \) and the degree of capital utilization \( z_t \). As stressed by eq. (7), the degree of capital utilization is a positive function of the rental rate of capital, \( z_t = z_1 r_t^k \), where \( z_1 = (1 - \psi)/\psi \) and \( \psi \) is a positive function of the elasticity of the capital utilization adjustment cost function normalized to belong to the [0,1] domain. Eq. (8) describes
the accumulation of installed capital $k_t$, featuring the convolutions $k_1 = (1 - \delta) / \gamma$ and $k_2 = [1 - (1 - \delta) / \gamma] (1 + \beta \gamma^{1 - \sigma_e}) \gamma^2 \varphi$. Installed capital is a function not only of the flow of investment but also of the relative efficiency of these investment expenditures as captured by the investment-specific technology disturbance $\xi_t$, which follows an autoregressive, stationary process. Eq. (9) relates to the monopolistic competitive goods market. Cost minimization by firms implies that the price mark-up $\mu^p_t$, defined as the difference between the average price and the nominal marginal cost or the negative of the real marginal cost, is equal to the difference between the marginal product of labor and the real wage $w_t$, with the marginal product of labor being itself a positive function of the capital-labor ratio and total factor productivity. Profit maximization by price-setting firms gives rise to the New-Keynesian Phillips curve, i.e., eq. (10), with the convolutions being $\pi_1 = t_p (1 + \beta \gamma^{1 - \sigma_e} t_p)^{-1}$, $\pi_2 = \beta \gamma^{1 - \sigma_e} (1 + \beta \gamma^{1 - \sigma_e} t_p)^{-1}$, and $\pi_3 = 1 / (1 + \beta \gamma^{1 - \sigma_e} t_p) [(1 - \beta \gamma^{1 - \sigma_e} \xi_p) (1 - \xi_p) / (\xi_p ((\phi_p - 1) \varepsilon_p + 1))]$. Notice that, in maximizing their profits, firms have to face price stickiness à la Calvo (1983). Firms that cannot reoptimize in a given period index their prices to past inflation. In equilibrium, inflation $\pi_t$ depends positively on past and expected future inflation, negatively on the current price mark-up, and positively on a price mark-up disturbance $\varepsilon^p_t$. The price mark-up disturbance is assumed to follow an ARMA(1,1) process. The inclusion of the MA term is to grab high-frequency fluctuations in inflation. When the degree of price indexation $t_p = 0$, $\pi_1 = 0$ and eq. (10) collapses to the purely forward-looking standard new-Keynesian Phillips curve. The assumption that all prices are indexed to either lagged inflation or trend inflation ensures the verticality of the Phillips curve in the long run. The speed of adjustment to the desired mark-up depends, among others, on the degree of price stickiness $\xi_p$, the curvature of the Kimball goods market aggregator $\varepsilon_p$, and the steady-state mark up, which in equilibrium is itself related to the share of fixed costs in production ($\phi_p - 1$) via a zero-profit condition. In particular, when all prices are flexible ($\xi_p = 0$) and the price mark-up shock is zero at all times, eq. (10) reduces to the familiar condition that the price mark-up is constant, or equivalently that there are no fluctuations in the wedge between the marginal product of labor and the real wage. Cost minimization by firms also implies that the rental rate of capital is negatively related to the capital-labor ratio and positively to the real wage (both with unitary elasticity) (see eq. (11)). Similarly, in the monopolistically competitive labor market, the wage mark-up will be equal to the difference between the real wage and the marginal rate of substitution between working and consuming, an equivalence captured by eq. (12), where $\sigma$ is the elasticity of labor supply with
respect to the real wage and \( \lambda \) is the habit parameter in consumption. Eq. (13) shows that real wages adjust only gradually to the desired wage mark-up due to nominal wage stickiness and partial indexation, the convolutions related to this equation being:

\[
\begin{align*}
    w_1 &= (1 + \beta \gamma^{1-\sigma_c})^{-1}, \\
    w_2 &= (1 + \beta \gamma^{1-\sigma_c} \xi_w)(1 + \beta \gamma^{1-\sigma_c})^{-1}, \\
    w_3 &= \xi_w(1 + \beta \gamma^{1-\sigma_c})^{-1}, \text{ and} \\
    w_4 &= 1/(1 + \beta \gamma^{1-\sigma_c}) \left(1 - \beta \gamma^{1-\sigma_c} \xi_w \right)(1 - \xi_w) / ((\phi_w - 1) \xi_w + 1)).
\end{align*}
\]

Notice that if wages are perfectly flexible \((\xi_w = 0)\), the real wage is a constant mark-up over the marginal rate of substitution between consumption and leisure. When wage indexation is zero \((\xi_w = 0)\), real wages do not depend on lagged inflation. Notice that, symmetrically with respect to the pricing scheme analyzed earlier, also the wage-mark up disturbance follows an ARMA(1,1) process. The model is closed by eq. (14), which is a flexible Taylor rule postulating a systematic reaction by policymakers to current values of inflation, the output gap, and output growth. In particular, one of the objects policymakers react to is the output gap, defined as a difference between actual and potential output (in logs). Consistently with the DSGE model, potential output is defined as the level of output that would prevail under flexible prices and wages in the absence of the two mark-up shocks. Then, policymakers engineer movements in the short-run policy rate \(r_t\), movements which happen gradually given the presence of interest rate smoothing \(\rho\). Stochastic departures from the Taylor rate, i.e. the rate that would realize in absence of any policy rate shocks, are triggered by a stochastic AR(1) process. Finally, eqs. (15)-(18) define the stochastic processes of the model, which features, as already pointed out, seven shocks (total factor productivity, investment specific technology, risk premium, exogenous spending, price mark-up, wage mark-up, and monetary policy). The model features a deterministic growth rate driven by labor-augmenting technological progress.

Readers seeking for further details on this framework are recommended to refer to the original *American Economic Review* 2007 article by Smets and Wouters (2007).

## 2 Bayesian estimation

To perform our Bayesian estimations we employed DYNARE, a set of algorithms developed by Michel Juillard and collaborators (Adjemian, Bastani, Juillard, Mihoubi, Perendia, Ratto, and Villemot (2011)). DYNARE is freely available at the following URL: http://www.dynare.org/.

The simulation of the target distribution is basically based on two steps.

- First, we initialized the variance-covariance matrix of the proposal distribution and employed a standard random-walk Metropolis-Hastings for the first \(t \leq t_0 = \ldots\)
20,000 draws. To do so, we computed the posterior mode by the "csminwel" algorithm developed by Chris Sims. The inverse of the Hessian of the target distribution evaluated at the posterior mode was used to define the variance-covariance matrix $C_0$ of the proposal distribution. The initial VCV matrix of the forecast errors in the Kalman filter was set to be equal to the unconditional variance of the state variables. We used the steady-state of the model to initialize the state vector in the Kalman filter.

- Second, we implemented the "Adaptive Metropolis" (AM) algorithm developed by Haario, Saksman, and Tamminen (2001) to simulate the target distribution. Haario, Saksman, and Tamminen (2001) show that their AM algorithm is more efficient than the standard Metropolis-Hastings algorithm. In a nutshell, such algorithm employs the history of the states (draws) so to "tune" the proposal distribution suitably. In particular, the previous draws are employed to regulate the VCV of the proposal density. We then exploited the history of the states sampled up to $t > t_0$ to continuously update the VCV matrix $C_t$ of the proposal distribution. While not being a Markovian process, the AM algorithm is shown to possess the correct ergodic properties. For technicalities, see Haario, Saksman, and Tamminen (2001).

We simulated two chains of 200,000 draws each, and discarded the first 90% as burn-in. To scale the variance-covariance matrix of the chain, we used a factor so to achieve an acceptance rate belonging to the [23%,40%] range. The stationarity of the chains was assessed via the convergence checks proposed by Brooks and Gelman (1998). The region of acceptable parameter realizations was truncated so to obtain equilibrium uniqueness under rational expectations.

As in Smets and Wouters (2007), we implement the theoretical restriction on the growth rate of the first four variables implied by the common quarterly trend growth rate of the labor-augmenting technological process, i.e.,

$$
\begin{bmatrix}
  \frac{\text{d} \text{GDP}_t}{\text{d}t} \\
  \frac{\text{d} \text{CONS}_t}{\text{d}t} \\
  \frac{\text{d} \text{INV}_t}{\text{d}t} \\
  \frac{\text{d} \text{WAG}_t}{\text{d}t} \\
  \frac{\text{d} \text{HOURS}_t}{\text{d}t} \\
  \frac{\text{d} \text{P}_t}{\text{d}t} \\
  \frac{\text{d} \text{FEDFUNDS}_t}{\text{d}t}
\end{bmatrix} =
\begin{bmatrix}
  \gamma_1 \\
  \gamma_2 \\
  \gamma_3 \\
  \gamma_4 \\
  \gamma_5 \\
  \gamma_6 \\
  \gamma_7
\end{bmatrix}
+ 
\begin{bmatrix}
  y_t - y_{t-1} \\
  c_t - c_{t-1} \\
  i_t - i_{t-1} \\
  w_t - w_{t-1} \\
  l_t \\
  \pi_t \\
  r_t
\end{bmatrix}.
$$
Some of the parameters are difficult to estimate, due to identification issues which are well known as for dynamic-stochastic rational-expectations models like this one (Canova and Sala (2009)). As in Smets and Wouters (2007), we then set the depreciation rate $\delta = 0.025$; the exogenous spending-GDP ratio $g_y = 0.18$; the steady-state mark-up in the labor market $\lambda_w = 1.5$; the curvature parameters of the aggregators in the goods and labor market $\varepsilon_p$ and $\varepsilon_w$ to 10. Tables 1 and 2 in the main text report our prior densities, which are carefully discussed in the original Smets and Wouters’ (2007) paper.

3 Further results

We document here the outcomes of some exercises which are commented in the main text of the paper but now shown there. Figure A1 plots our impulse responses obtained with actual U.S. data when employing the growth rate of output in place of the CBO output gap in our trivariate VARs. Figure A2 depicts the outcome of our Monte Carlo exercise conditional on trivariate VARs embedding the growth rate of output in place of the model-consistent output gap. Figure A3 reports our robustness checks dealing with i) VAR(10) models or ii) VAR(3) models estimated with 1,000 simulated data. Figure A4 reports the outcome of our Monte Carlo exercise conditional on our posterior means estimated with 1966Q1-1979Q3 data. Finally, Figure A5 shows the outcome of our Monte Carlo exercises focusing on sign restriction-VARs conditional on different calibration of the monetary policy shock’s standard deviation for the DGP in a context in which no restrictions on the response of output are imposed.
Figure A2. **DSGE-** vs. **CVAR-impulse response functions to a monetary policy shock:** 1984Q1-2008Q2 – Output expressed in growth rates. Dashed-circled red lines: DSGE Bayesian mean impulse responses. Dashed black lines: Cholesky-VAR mean responses. Shaded areas: Cholesky-VAR responses, [16th,84th] percentiles. Simulations based on 1,000 repetitions of our Monte Carlo algorithm. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: inflation, output gap, nominal rate). VAR estimated with a constant and three lags.
Figure A3. Monte Carlo exercises, Cholesky-VAR mean responses: Robustness checks. Black lines: Baseline scenario. Blue diamonded-line: results conditional on VARs estimated with 10 lags. Magenta lines with squares: Results conditional on VARs estimated with 1,000 observations. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: inflation, output growth, federal funds rate). VAR estimated with a constant and three lags when not otherwise specified.
Figure A5. **DSGE- vs. Sign Restriction-VAR-impulse response functions to a monetary policy shock, 1984Q1-2008Q2: No restrictions on the response of output.** Dashed-circled red lines: DSGE Bayesian mean impulse responses. Other lines: Sign Restriction-VAR impulse responses à la Fry and Pagan (2011) computed over 1,000 retained draws and conditional on different calibrations of the standard deviation of the monetary policy shock in the Smets-Wouters (2007) model used as DGP (estimated standard deviation, twice the estimated standard deviation, and five times the estimated standard deviation). VARs estimated with a constant and three lags.
References


